# Department of Statistics

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## Question Bank for Part – I (Hons)

## PAPER I

1. State the classical definition of probability. Discuss its limitations, giving examples. 2

2. For any two events A and b, show that P (AUB) $\geq$ max {P (A), P (B)}. 2

3. Find the probability that a leap year, selected at random, will have 53 Saturdays.

4. Define moment generating function (m.g.f.) of a random variable. Give example of a distribution whose m.g.f. does not exist. Obtain the m.g.f. of a Poisson distribution with parameter $\theta$. Hence obtain the mean of the distribution. ( 2 + 2 + 3 + 3 )

5. For any two events show that P (AUB) = P(A) + P(B) + P ( A$\cap$B). Hence show that P ( A$\cap$B) $\geq$ P(A) + P(B) – 1. 3

6. If A and B are independent, then show that $A^c$ and B are also independent. 2

7. n cheques drawn in favour of n suppliers are randomly placed in n envelopes addressed to the suppliers. What is the probability that no cheques goes into the right envelope? Also calculate the limiting value of this probability when $n \to \infty$. 5

8. If X has the geometric distribution with parameter p, show that $P(X \geq m + n / X \geq n) = P(X \geq n)$, m,n ε N. Interpret the result.

9. For a r.v. $X \sim N(\mu, \sigma^2)$, prove that $\mu_{2r} = (2r-1)(2r-3)\ldots\ldots 5.3.1\, \sigma^{2r}$ where $\mu_{2r}$ is the central moment of order 2r. 5

10. Consider the r.v. X with p.d.f. $f(x) = b \exp[-b(x-a)]$; $a < x < \infty$, $b>0$. Show that $E|X - \mu_e| = \log 2\, e / b$. [$\mu_e$ = median]. 5

11. If $X \sim B(2s, ½)$ then show that $1/2\sqrt{s} < P(X = s) < 1/\sqrt{(2s+1)}$. 5

12.. Define *'vector space'* and *'basis'* of a vector space. If ( $\boldsymbol{\alpha_1, \alpha_2, \ldots\ldots, \alpha_r}$ ) and ($\boldsymbol{\beta_1, \beta_2, \ldots\ldots, \beta_s}$) be different bases of a vector space , show that r = s. What is meant by orthogonal basis ? 4 + 4 + 2

13. State and prove Lagrange's mean-value theorem. 3

14. Examine the differentiability of $f(x) = |x|$. 2

15. Show that the determinant of a skew-symmetric matrix of odd order is zero. 2

16. Find the eigenvalues of a matrix $A^{n \times n}$ where $A^2 = A$. 3

17. Write down whether the following statements are ***true*** or ***false***: 1 (each)

   a) By the classical definition of probability it is meant that the probability of an event is any real number between 0 and 1, both exclusive.
   b) If P(A) = 0, then P(A$\cap$B) = 0.
   c) If two events A and B are mutually exclusive then P(AUB) = P(A) + P(B) .
   d) A set of vectors containing the null vector is linearly independent.
   e) Lt x sin(1/x) does not exist.

18 . Answer the following : 2 (each )

a) What are the limitations of classical definition of probability ?

b) If $P(A) = P(B) = 1$ , find $P(A \cap B)$.

c) If $P(A) = 1/3$ and $P(B^c) = 1/4$ , can A and B disjoint ? Explain.

d) Two digits are chosen at random without replacement from the set of integers { 1, 2, 3, 4, 5 }. Find the probability that both the digits are greater than 3.

e) A card is drawn from a well shuffled pack of 52 cards. What is the probability that it is either an ace or a red card ?

f) A problem in statistics is given to two students A and B whose chances of solving it are ½ and ¾ respectively. What is the probability that the problem is solved ?

g) Evaluate : Lt $x^{-1} \sin x^0$ .

h) Examine the differentiability of $f(x) = | x |$.

i) Find the angle between two non-zero vectors.

j) Define linear independence of a set of vectors.

19. Answer the following :

a) Show that $x/(1 + x) < \log(1 + x) < x$, if $x > 0$. 6

b) n cheques are drawn in favour of n suppliers are randomly placed in n envelopes addressed to the suppliers. What is the probability that no cheques goes into the right envelope ? Also calculate the limiting value of this probability as $n \to \infty$.

20 . Give the *axiomatic definition* of probability. Hence show that $\mathbf{P(\phi)} = 0$. 3 + 2 = 5

21. For n events $A_1, A_2, \ldots\ldots, A_n$ show that $P(\cap A_i) \geq \Sigma P(A_i) - (n - 1)$. 5

22. If X has the geometric distribution with parameter p, show that $P(X \geq m + n / X \geq n) = P(X \geq n)$, $m,n \in N$. Interpret the result. 5

23. If a r.v. X has the p.d.f. $f(x) = \theta e^{-\theta x}$, $0 < x < \infty$, then find $E(X / X > a)$ and $V(X / X > a)$. 5

24. For any two events A and B, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Hence show that $P(A \cap B) \geq P(A) + P(B) - 1$. 3 + 2 = 5

25. Define conditional probability. When two events A and B are said to be independent ? If A and B are independent, then show that $A^c$ and $B^c$ are also independent. 1 + 1 + 3 = 5

26. Find the probability that a randomly selected year will have **53** *Sundays*. 3

27. Define distribution function of a r.v. X. State its important properties. Suppose $F_1$ and $F_2$ are distribution functions. If α and β are non-negetive numbers, whose sum is unity, show that $\alpha F_1 + \beta F_2$ is also a distribution function. ( 1 + 3 + 3 )

28. For a r.v. $X \sim N(\mu, \sigma^2)$, prove that $\mu_{2r} = (2r - 1)(2r - 3)\ldots\ldots 5.3.1\, \sigma^{2r}$ where $\mu_{2r}$ is the central moment of order 2r. 5

29. Consider the matrix $A = (1 - \rho)\, \mathbf{I_n} + \rho\, \mathbf{1\,1'}$ where $\mathbf{1'} = (1, 1, 1, \ldots\ldots, 1)^{1xn}$. Evaluate $| A |$. Show that A is positive definite iff $-1 / (n - 1) < \rho < 1$. 5

30. Define *'vector space'* and *'basis'* of a vector space. If ( $\boldsymbol{\alpha_1, \alpha_2, \ldots\ldots, \alpha_r}$ ) and ($\boldsymbol{\beta_1, \beta_2, \ldots\ldots, \beta_s}$ ) be different bases of a vector space , show that r = s. What is meant by orthogonal basis ? ( 4 + 4 + 2 )

31. . a) State and prove Lagrange's mean-value theorem.

b) Examine the differentiability of $f(x) = |x|$.

c) Show that the determinant of a skew-symmetric matrix of odd order is zero.

d) Find the eigenvalues of a matrix $A^{n \times n}$ where $A^2 = A$. ( 3 + 2 + 2 + 3

# PAPER II

1. Examine the characteristics given below and state, in each case, whether it is an attribute or a variable. In the latter case indicate whether it is discrete or continuous :
   mother tongue, annual income of a family and number of workers of a rice mill. 3
2. Define class boundary and cumulative frequency . 2
3. What do you mean by 'ogives' ? How are they constructed ? Mention their uses. 5
4. Describe the different parts of a table. 4
5. What is skewness ? Suggest a measure of skewness based on quartiles. Show that this measure lies between -1 and +1. 6
6. What is ratio chart ? Point out its advantage over a simple line diagram. 2 + 2
7. Show that mean deviation about the median cannot be greater than the standard deviation. 4
8. Find the variance of first n natural numbers. 2

9. For two attributes having two categories each, explain the ideas of independence and association, and hence those of complete and absolute association. 5
10. Let s and R, respectively denote the *standard deviation* and *range* of a set of n values of a variable. Then show that $R^2 / 2n \le S^2 \le R^2 / 4$. When do the equalities hold ? 5
11. In a joint study of two variables x and y, paired data on ( x, y ) have been obtained. On the basis of such data obtain a) a LS regression equation of x on y,
    b) the variance of the predicted x-values and
    c) the residual variance. ( 6 + 2 + 2 )
12. State one situation when rank correlation may be used. Derive Spearman's measure of rank correlation coefficient for an untied case. Discuss the cases when it can assume the extreme values. ( 2 + 4 + 4 )

13. Define intra-class correlation coefficient. Derive the formula for intra-class correlation coefficient ($R_I$) when a variate x is measured for p families, each containing k members. Obtain the limits of $R_I$ with their interpretation. ( 2 + 4 + 4 )
14. Answer the following : 1 (each)

a) Mention a method of collecting ***primary data***.
b) When do you prefer a ***ratio chart*** to represent data ?
c) Define ***relative*** frequency.
d) Define ***co-efficient of variation*** (C.V.).
e) Mention a disadvantages of ***harmonic mean*** (H.M.).
f) What is meant by ***median*** of a distribution ?

15. Answer the following : 2 (each )

a) Distinguish between an ***attribute*** and a ***variable***.
b) How is ***Histogram*** drawn for a grouped frequency distribution ?
c) Define an ***ogive***. How is it drawn ?
d) Give an example when **H.M.** is more appropriate compared to **A.M.**
e) Write down the formula for ***mode*** in case of a grouped frequency distribution.

f) If A, G and H be the A.M., G.M. and H.M. respectively of two positive numbers **m** and **n** then show that $G^2 = AH$.

g) Find the variance of **1st n** natural numbers.

h) Give suitable measure of ***central tendency*** and measure of ***dispersion*** for a frequency distribution
with both the terminal classes are open.

i) Show that ***standard deviation*** is independent of any change of origin but dependent on the change
of scale.

j) Give a measure of ***skewness*** in terms of 2nd and 3rd order central moments.

k) Show that $[(n + 1)/2]^n \geq n!$, where n is positive integer.

l) The numbers 3.2, 5.8, 7.9, and 4.5 have frequencies x, (x+2), (x-3) and (x+6) respectively. If their A.M.
is 4.876, find the frequencies.

16 . Prove that $A \geq G \geq H$ with usual notations. 6

17 What is ***Skewness*** ? Suggest a measure of ***Skewness*** based on quartiles.
Show that it lies between **-1** and **+1** 2 + 2 + 2.

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## Question Bank for Part – II (Hons)

## PAPER IV

Problem : 1

1. Show that V(X) = E [ V ( X/Y ) ] + V [ E ( X/Y ) ]. 5
2. If the regression of y on x is linear and the conditional variance $\sigma^2_{y.x}$ = V(Y/X) is independent of X then show that $\sigma^2_{y.x} = \sigma^2 y\,( 1 - \rho^2 )$. 5
3. State and prove Chebyshev's WLLN. 5
4. Let { Xn } be a sequence of random variables, where Xn = 0 with probability 1/n, = 1 with probability (1- 1/n). Show that Xn converges in probability to 1. 5
5. Define multiple regression and multiple correlation. Show that if $\rho_{1j} = 0$, j = 2,3,……,p then $\rho_{1.23....p} = 0$ . 6
6. Derive Simpson's 1/3rd rule for an integrable function over a finite interval (a, b). 4
7. Show that E ≡ 1 + Δ. 3
8. If $\rho_{1j} = \rho$ ( j = 2,3,……,p ) and $\rho_{ij} = \rho'$ (i, j = 2,3,……,p ; i ≠ j ) then find the values of $\rho_{1.23....p}$ and $\rho_{12.3....p}$.
9. If $x_1, x_2, \ldots, x_k$ have a multinomial distribution with parameters n; $p_1, p_2, \ldots, p_k$ , find the m.g.f. of the distribution. Hence find the means, variances and co-variances. 8
10. Prove that Δ log x = log [ 1 + {Δf(x) / f(x)} ]. 2
11. Derive Lagrange's interpolation formula and show that the sum of the Lagrangian co-efficients is unity. 7
12. Given f(0) = 0, f(1) + f(2) = 10 and f(3) + f(4) + f(5) = 65 ; find f(4). 3
13. A trivariate normal distribution has the p.d.f. $f(x_1, x_2, x_3) = k.\, e^{-\frac{1}{2} Q(x1,\, x2,\, x3)}$ where $Q(x_1, x_2, x_3) = 7x_1^2 + 4x_2^2 + 2x_3^2 + 6x_1x_2 + 4x_1x_3 + 2x_2x_3 - 8x_1 + 2x_2 - 2x_3 + 5$. Find the mean-vector and the generalized variance of this distribution. 5
14. When a function f is said to be Reimann integrable ? Let f(x) = 0, when x is rational ; = 1, when x is irrational. Show that f is not integrable in any interval. ( 5 + 5 )
15. State Chebyshev's inequality. Hence or otherwise prove that P( | X – 2 | > 2 ) ≤ ½ , if X has the p.m.f. $f(x) = 2^{-x}$ , x = 1,2,3,…….. ( 2+ 3 )
16. Let { $X_n$ } be a sequence of bounded random variables. Show that, as **n → ∞** , $X_n$ ---P----> X ==> $X_n^{\,2}$----P----> $X^2$. 5
17. Show that V(X) = E [ V ( X/Y ) ] + V [ E ( X/Y ) ]. 5
18. If the regression of y on x is linear and the conditional variance $\sigma^2_{y.x}$ = V(Y/X) is independent of X then show that $\sigma^2_{\mathbf{y.x}} = \sigma^2_{y}\,( 1 - \rho^2 )$. 5 (
19. Show that E ≡ 1 + Δ.
20. Suppose M is an operator defined by M u(x) = M u(x – h) + u (x – h/2), h being the unit of differencing. Express M in terms of ordinary operators Δ and E. 4
21. State and prove the Fundamental Theorem of Finite Differences. 6
22. If $x_1, x_2, \ldots, x_k$ have a multinomial distribution with parameters n; $p_1, p_2, \ldots, p_k$ , find the m.g.f. of the distribution. Hence find the means, variances and co-variances. 8
23. Prove that Δ log x = log [ 1 + {Δf(x) / f(x)} ]. 2

24. Given f(0) = 1, f(1) + f(2) = 10 and f(3) + f(4) + f(5) = 65 = f(5). Find the function f(x). 4

25. Define a factorial power function $x^{(n)}$. Prove that $\Delta x^{(n)} = nh\, x^{(n-1)}$, h = interval of differencing.

Express $3x^3 - 4x^2 + 3x - 11$ in terms factorial notation. ( 2 + 2 + 2 ) )

26. Use the method of separation of symbol to prove

$u_0 + {}^xC_1 \Delta u_1 + {}^xC_2 \Delta^2 u_2 + {}^xC_3 \Delta^3 u_3 + \ldots\ldots = u_x + {}^xC_1 \Delta^2 u_{x-1} + {}^xC_2 \Delta^4 u_{x-2} + \ldots\ldots$ 5

27. When a function f is said to be Reimann integrable ?

Let f(x) = 0, when x is rational ; = 1, when x is irrational. Show that f is not integrable in any interval.
5

28. a) With the help of the following table, determine the value of θ for which ***sin ϑ*** = **0.75** using

a suitable interpolation formula. 5

| ***ϑ*** | $47^0$ | $48^0$ | $49^0$ | $50^0$ | $51^0$ |
|---|---|---|---|---|---|
| ***Sin ϑ*** | 0.73135 | o.74314 | 0.75471 | 0.76604 | 0.77715 |

b) Evaluate the value of cube root of 28 to three decimal places by the Newton-Raphson method

of numerical solution of equation. 5

c) The angle of rotation (radians) of a rod is given for various values of time t in seconds. Find the

angular velocity when t = 1.2 seconds. 5

| ***t*** | 0.0 | 0.4 | 0.8 | 1.2 |
|---|---|---|---|---|
| ***ϑ = f(t)*** | 0.000 | 0.493 | 2.022 | 4.666 |

29. a) Show that if a number is corrected to n significant figures, then the relative error is less than

$K^{-1}.10^{-(n-1)}$ where k is the 1st significant figure in the number. 6

b) What do you mean by Δ and E operators? Show that E . Δ ≡ Δ . E. ( 3 + 2 ) = 5

c) Show that $D = h^{-1} (\Delta - ½ \Delta^2 + 1/3\ \Delta^3 - ¼ \Delta^4 + \ldots\ldots)$ where D is the differential operator. 4

30. a) Use Euler-Maclaurin's sum formula to prove that 5

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{4a} \left(1 + \frac{1}{6a}\right) + \sum_{x=1}^{a} \frac{a}{a^2+x^2}$$

b) For a linear transformation x = α + βt and $x_i = \alpha + \beta t_i$, i = 0, 1, 2, ….,n ; if f(x) = F(t), then show that

$f(x_0, x_1, \ldots, x_n) = \beta^{-n} F(t_0, t_1, \ldots, t_n)$ where $g(x_0, x_1, \ldots, x_n)$ denotes the $n^{th}$ order divided

difference of arguments $x_0, x_1, \ldots, x_n$. 5

c) Explain the principle of numerical differentiation. Deduce Newton's backward differentiation formula for 1st and 2nd order differentiation. 5

31 a) What do you mean by numerical integration ? Derive Simpson's 1/3rd and Simpson's 3/8th rules for an integrable function over a finite interval (a, b). Suggest how the value of **π** can be calculated using Simpson's 1/3rd rule. 10

b) Show that the sum of the coefficients of entries in Lagrange's formula is unity. 5

32. a) State Stirling's and Bessel's interpolation formula. When are they used ? State, in particular, Bessel's formula for interpolating two halves. 5

b) Establish superiority of the Simpson's 1/3rd rule over Simpson's 3/8th rule in respect of their computational accuracy. 5

c) Find the remainder in approximating f(x) by interpolation polynomial using distinct nodes $x_0, x_1, x_2, \ldots\ldots, x_n$. 5

33. a) Let $y = ax^2 + bx + c$ be the equation of the parabola passing through $( -h, y_0 )$, $( 0, y_1 )$ and $( h, y_2 )$. Find the area underlying the parabola bounded by the x-axis and two ordinates at – h and h using Simpson's 1/3rd rule. Show that, the calculated value agrees with that obtained by exact value and Indicate the reason behind it. 5

b) Using Euler-Maclaurin's sum formula and the limiting value of the Walli's product, derive Stirling's approximation for factorials. 5

c) Describe the method of Iteration for solving an equation involving one unknown variable. Also derive the condition under which the process converges.

34. a) Find by the method of iteration the positive root of the equation $e^x = 1 + 2x$, correct to 5 places of decimals. 5

b) Evaluate the following by Simpson's 1/3rd and 3/8th rule (correct to 4 decimal places). Also obtain the errors of approximation in each cases. 5

$$\int_0^1 \frac{dx}{1 + x^2}$$

b) Apply a suitable numerical method to find the value of $\log_e 3$. 5

35. a) Suppose **B** is an operator defined by

$$\mathbf{B}\, f(x) = \mathbf{B}\, f(x - h) + f(x - h/2),$$

h being the unit of differencing. Express **B** in terms of ordinary operators **Δ** and **E** . 2

b) What do you mean by forward and backward difference operators. How are they related ?
2 + 2 = 4

c) State and prove the ***fundamental theorem of finite differences***. 5

d) By applying method of separation of symbols, establish the following identity :

$$u(x) - {}^nC_1\, u(x-h) + {}^nC_2\, u(x-2h) - \dots\dots + (-1)^n\, {}^nC_n\, u(x-nh) = n!\, h^n ,$$

where u(x) is a polynomial of degree n in x and h is the unit of differencing. 4

36. a) Define factorial power function $x^{(n)}$. Prove that $\boldsymbol{\Delta}\, x^{(n)} = nh\, x^{(n-1)}$, h being interval of differencing. Express a polynomial of degree n in factorial notations. 1+2+4 = 7

b) Express $n^{th}$ order divided difference in terms of ordinary difference operator. Establish Newton's divided difference interpolation formula. 4 + 4 = 8.

37. a) Explain the term 'Inverse Interpolation'. Give an outline of the method of successive approximations for inverse Interpolation. 5

b) State Stirling's interpolation formula. Using it show that

$$f'(x) = 2/3\, [\, f(x+1) - f(x-1)\,] - 1/12\, [\, f(x+2) - f(x-2)],$$

considering the differences up to $3^{rd}$ order. 5

c) If $u_x = a + bx + cx^2$, then show that $\int u_x\, dx = 2\, u_2 + 1/12\, (\, u_0 - 2u_2 + u_4\, )$. Hence find an approximate value of

$$\int \exp(-x^2/10)\, dx.$$ 5

38. a) Describe Newton- Raphson method for solving an equation of one unknown variable. Obtain the condition of convergence of the solutions. 5

b) State the rules of rounding of approximate numbers with examples. If a number 0.000012 is approximated to 0.000009, find the number of significant digits for such approximation. 5

c) Evaluate the limiting value of the Walli's product. 5

39. . a) Derive the ***Trapizoidal*** rule for numerical integration. Can you suggest an improved way of numerical integration ? 4 + 2 = 6

b) State and prove ***Euler-Maclaurin's*** sum formula and hence obtain the sum of $4^{th}$ power of the $1^{st}$ n natural integers. 5 + 4 = 9.

## PAPER V

1. What is sampling distribution ? Define Jacobian of transformation in case a one-to-one transformation. Let $X_1$ and $X_2$ are two independent binomial variables with parameters ($n_1$, p) and ($n_2$, p) respectively. Find the distribution of $X_1/( X_1 + X_2 = t )$. ( 2 + 2 + 6)

2. What are the criterion of a good estimator ? What are the sufficient conditions for consistency ? Define an unbiased estimator. Show that the sample mean is an unbiased and consistent estimator of the population mean. (2+1+1+ 3 + 3 )
3. Define an MLE. Mention three important properties of an MLE. In sampling from a $N(\mu,\sigma^2)$, where $\mu$ and $\sigma^2$ both are unknown, find the MLE of $\mu$ and $\sigma^2$. Check whether these MLEs are unbiased or not. ( 1 + 2 + 5 + 2 )
4. Define a statistical hypothesis. When it is called simple or composite ? Give an example. Distinguish between level and size of a test. What is the p-value of a test ? The p-value of a test statistic is 0.001. Will you accept the null hypothesis at 5% level of significance. ( 2 + 2 + 2 + 2 + 2 )

5. Define an unbiased test. Show that an UMP test is necessarily unbiased. Based on independent observations from a $N(\mu,\sigma^2)$ population with known $\sigma^2$ , derive the UMP size $\alpha$ test for $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$. (2 + 2 + 6 )
6. Let X and Y be B(m, ½) and B(n, ½) variables respectively. Find the distribution of ( X – Y + n ). What happens if probability of success ≠ ½. 6
7. If $X \sim R(0,1)$, find the distribution of $U = - 2 \log_e X$. 4

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from $R(0, \theta )$ distribution. Let $X_{(n)} = \text{Max } X_i , 1 \le I \le n$. Find the distribution of $X_{(n)}$. Also find its mean. 5
9. If $\rho = 0$ then show that $t = r ( n - 2 )^{-1/2} / ( 1 - r^2 )^{-1/2}$ follows t-distribution with (n-2) d.f. where $\rho$ and r are population and sample correlation coefficient respectively. 5
10. For a random vector $\mathbf{X} = ( X_1, X_2, \ldots, X_m )'$ having multivariate distribution with $E ( \mathbf{X} ) = \boldsymbol{\mu}$ and $Disp( \mathbf{X} ) = \Sigma = (( \sigma_{ij} ))_{m \times m}$, justify that the multiple correlation ( $\rho_{1.23\ldots m}$ )between $X_1$ and ( $X_2, \ldots, X_m$ ) is always greater than the correlation between $X_1$ and any linear combination of $X_2, \ldots, X_m$. 5
11. Suppose $( X_1 , X_2 ) \sim BN ( 0, 0, 1, 1, \rho )$. Show that $V( X_1^2 + X_2^2 - 2\rho X_1 X_2 ) = 4 ( 1 - \rho^2 )^2$. 5

Problem : 12

What is sampling distribution ? Define Jacobian of transformation in case a one-to-one transformation. Let $X_1$ and $X_2$ are two independent $P(\theta)$. Find the distribution of $X_1/( X_1 + X_2 = t )$. ( 2 + 2 + 6 )

Problem : 13

Citing example , define an estimator and an estimate. Define sufficient statistics. Suppose $x_1, x_2, \ldots, x_n$ be a random sample of size n from B(1, p) population. Find sufficient statistic for p. ( 3 + 2 + 5 )

Problem : 14

State and prove ( only for discrete case ) Fisher-Neyman factorization theorem for sufficiency. Find sufficient statistics for the parameters ( $\alpha$, $\beta$ ) on the basis of a random sample drawn from a distribution with p.d.f. $f(x) = \beta e^{-\beta(x-\alpha)}$ ; $x > \alpha$; $\alpha,\beta > 0$. ( 5 + 5 )

Problem : 15

Define an unbiased estimator. Does it always exist ? Give example. Show that the sample mean is an unbiased estimator of the population mean. Giving an example show that an unbiased estimator is not necessarily unique. ( 2 + 2 + 3 + 3 )

Problem : 16

i) Define UMVUE of a parametric function $g(\theta)$. Show that, if exists, it is unique.

ii) Define an MVBE. Show that an MVUE may not necessarily be an MVBE. Is the converse always true ? (5+5)

Problem :17

a) If $X \sim B(n, p)$ then what will be the distribution of $Y = n - X$ ?

b) Let X and Y be $B(m, ½)$ and $B(n, ½)$ variables respectively. Find the distribution of ( $X - Y + n$ ).

c) If $X \sim R(0,1)$, find the distribution of $U = -2 \log_e X$. ( 2 + 4 + 4 )

Problem : 18

a) Let X be an absolutely continuous random variable with p.d.f. $f(x)$. Then show that $1 - F(x) \sim R(0,1)$ , where $F(x)$ is the c.d.f. of X.

b) If the random X follows the standard Cauchy distribution, then find the distribution of $Y = X^2$.

c) Let $X_1$ and $X_2$ be independent random variables, each being distributed as $N(0, \sigma^2)$ . Define $Y_1 = \max(X_1, X_2)$, $Y_2 = \min(X_1, X_2)$. Obtain $E(Y_1)$ and $E(Y_2)$. Also show that $(Y_1 + Y_2)$ and $(Y_1 - Y_2)$ are independent. (3+3+4)

## Question Bank for Part – III (Hons)

PAPER VII

1. Explain crude and standardized death rates. In what way is standardized death rate superior to crude death rate? Define Stable population and stationary population. ( 4 + 2 + 4 )

2. What is expectation of life? Distinguish between curate expectation of life $e_x$ and complete expectation of life $e^0_x$ at age x and find an approximate relationship between them. Show that with usual notations i) $m_x = 2q_x / ( 2 – q_x )$ and ii) $p_x = e_x / ( 1 + e_{x+1} )$. ( 2 + 4 + 4 )

3. What is meant by $\mu_x$, the force of mortality at age x? Can this exceed one? Derive Makeham's graduation formula for $l_x$ starting from a suitable form of $\mu_x$. Distinguish between population estimates and population projections. ( 2+1+5+2 )

4. Give the general form of the equation of the Logistic curve and state its properties. Describe Rhodes' method for fitting a Logistic curve. ( 4 + 6 )

5. a) What do you mean by 'homogeneity error' in index number?
b) If $L_p$, $P_p$ & $L_q$ denote respectively Laspeyres' price index, Paasche's price index and Laspeyres' quantity index, show that $L_q ( P_p – L_p )$ may be looked upon as the weighted covariance between price relatives and quantity relatives, the weights being the base year values.
c)What is a chain index? ( 3 + 5 + 2 )

6. What do you mean by a time series? Discuss briefly the moving average method for determining trend in a time series. Define an AR(p) process and check whether it is weakly stationary. ( 1 + 5 + 4 )

7. a) What do you mean by 'price elasticity of demand'?
b) If the demand curve is of the form $p = a\, e^{-kx}$, where p is the price and x is the demand, prove that the elasticity of demand is 1 /kx. Hence deduce the elasticity of demand for the curve $p = 10\, e^{-x/2}$.
c) What is an Engel's curve and how will you determine it on the basis of family budget data. 2 + 3 + 5

8. Explain, with an example, the problem of forecasting in the context of time series analysis. Explain how the exponential smoothing technique can be used for forecasting. How this technique can be modified if there are trend and seasonal components in the series? ( 2 + 5 + 3 )

9. a) What do you mean by GDP of a country ?
b) Distinguish between Fixed base Index number and chain base Index number. Point out their relative merits and demerits specially in the light of different errors involved. ( 3 + 3 + 4 )

10. What do you mean by a time series? Discuss briefly the moving average method for determining trend in a time series. Define an AR(p) process and check whether it is weakly stationary. ( 1 + 5 + 4 )

11. a) What do you mean by 'price elasticity of demand'?
b) If the demand curve is of the form $p = a\, e^{-kx}$, where p is the price and x is the demand, prove that the elasticity of demand is 1 /kx. Hence deduce the elasticity of demand for the curve $p = 10\, e^{-x/2}$.
c) What is an Engel's curve and how will you determine it on the basis of family budget data. ( 2 + (2 + 1) + 5 )

12. Explain, with an example, the problem of forecasting in the context of time series analysis. Explain how the exponential smoothing technique can be used for forecasting. How this technique can be modified if there are trend and seasonal components in the series? ( 2 + 5 + 3 )

13. Explain crude and standardized death rates. In what way is standardized death rate superior to crude death rate? Define Stable population and stationary population. ( 4 + 2 + 4 )

14. What is expectation of life? Distinguish between curate expectation of life $e_x$ and complete expectation of life $e^0{}_x$ at age x and find an approximate relationship between them. Show that with usual notations i) $m_x = 2q_x / ( 2 - q_x )$ and ii) $p_x = e_x / ( 1 + e_{x+1} )$. ( 2 + 4 + 4 )

15. . What is meant by $\mu_x$, the force of mortality at age x? Can this exceed one? Derive Makeham's graduation formula for $l_x$ starting from a suitable form of $\mu_x$. Distinguish between population estimates and population projections. ( 2+1+5+2 )

16. Give the general form of the equation of the Logistic curve and state its properties. Describe Rhodes' method for fitting a Logistic curve. ( 4 + 6 )

17. a) What do you mean by 'homogeneity error' in index number?
b) If $L_p$, $P_p$ & $L_q$ denote respectively Laspeyres' price index, Paasche's price index and Laspeyres' quantity index, show that $L_q ( P_p - L_p )$ may be looked upon as the weighted covariance between price relatives and quantity relatives, the weights being the base year values.
c)What is a chain index? ( 3 + 5 + 2 )

18. What do you mean by a time series? Discuss briefly the moving average method for determining trend in a time series. Define an AR(p) process and check whether it is weakly stationary. ( 1 + 5 + 4 )

19. a) What do you mean by 'price elasticity of demand'?
b) If the demand curve is of the form $p = a\, e^{-kx}$, where p is the price and x is the demand, prove that the elasticity of demand is 1 /kx. Hence deduce the elasticity of demand for the curve $p = 10\, e^{-x/2}$.
c) What is an Engel's curve and how will you determine it on the basis of family budget data. 2+3+5

20. Explain, with an example, the problem of forecasting in the context of time series analysis. Explain how the exponential smoothing technique can be used for forecasting. How this technique can be modified if there are trend and seasonal components in the series? ( 2 + 5 + 3 )

21. Give the general form of the equation of the Logistic curve and state its properties. Describe Rhodes' method for fitting a Logistic curve. ( 4 + 6 )

22. What is expectation of life? Distinguish between curate expectation of life $e_x$ and complete expectation of life $e^0_x$ at age x and find an approximate relationship between them. Show that with usual notations i) $m_x = 2q_x / ( 2 - q_x )$ and ii) $p_x = e_x / ( 1 + e_{x+1} )$. ( 2 + 4 + 4 )

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24. Explain crude and standardized death rates. In what way is standardized death rate superior to crude death rate? Define Stable population and stationary population. ( 4 + 2 + 4 )

## **Paper : VIII**

Problem 1

a) When a linear-model is said to be an ANOVA model ? What is 'valid-error' in the context of ANOVA?
b) Discuss, in detail, the analysis of variance technique for one-way classified data under fixed effect model. Derive the expectation of between SS. (2 + 2 + 6)

Problem 2

a) In what respects do analysis of variance, regression analysis and analysis of covariance differ?
b) Stating appropriate assumptions, describe tests based on analysis of variance technique for
i) presence of regression and ii) linearity of regression
in a bivariate set-up where the independent variable is assumed to be non-stochastic. ( 3 + 7 )

Problem 3

a) What is a treatment contrast?
b) What are the basic principles of design of experiments?
c) Describe the layout and advantages of a RBD. d) Derive the efficiency of a RBD compared to a CRD. ( 1 + 3 + 3 + 3 )

Problem 4

a) What do you mean by a linear model? Explain how the model for one way classification can be considered as a linear model. b) What is local control ? Give an example. c) Workout the analysis of an RBD when two observations are missing. ( 3 + 2 + 5 )

Problem 5

a) Explain the terms 'complete confounding' and 'partial confounding'.
b) Identify the confounded effects of a replicate of $(2^5, 2^2)$ design in blocks of 8 plots, of which the following is a block : ( a, abc, ade, bd, be, cd, ce, abcde ).
c) Give in detail the analysis of a $2^3$-experiment conducted in an RBD. ( 2 + 2 + 6 )

Problem 6

a) A simple random sample of size 3 is drawn from a population of size N with replacement. Show that the probability that the sample contains 2 different units is $3(N - 1)/N^2$.
b) In SRSWOR, show that the sample proportion p is an u.e. of the population proportion P. Also find V(p) and v(p).
( 4 + 6 )

Problem 7

a) What are the advantages of sample survey over complete enumeration ?
b)If the population consists of a linear trend, then show that $V_{st} \leq V_{sys} \leq V_{rand}$ . ( 4 + 6 )

Problem 8

a) What is double sampling procedure ? Give an example where the double sampling procedure is used.
b) To get ratio estimator of the population mean double sampling procedure is adopted. Find out approximate variance of the estimator.

c) Introducing appropriate cost function, compare the performance in (b) with that of an estimator based on a single sampling procedure with a specified total cost. ( 2 + 4 + 4 )

Problem 9

a) What do you mean by linear model ? When it is called an ANOVA model?

b) Discuss, in detail, the analysis of variance technique for one -way classified data under fixed effects model. Derive the expectation SSE. (2 + 2 + 6)

Problem 10

a) What do you mean by degrees of freedom (df) and valid error?

b) Stating appropriate assumptions, describe tests based on analysis of variance technique for

i) presence of regression and ii) linearity of regression

in a bivariate set-up where the independent variable is assumed to be non-stochastic. ( 3 + 7 )

Problem 11

What are the basic principles of design of experiments? Describe the layout and advantages of a RBD. Derive the efficiency of a RBD compared to a CRD. ( 4 + 3 + 3 )

Problem 12

Explain how the model for one way classification can be considered as a linear model. What is local control ? Give an example. Workout the analysis of an RBD when two observations are missing. ( 2 + 2 + 6 )

Problem 13

a) Explain the terms 'complete confounding' and 'partial confounding'.

b) Identify the confounded effects of a replicate of ($2^5$, $2^2$) design in blocks of 8 plots, of which the following is a block : ( a, abc, ade, bd, be, cd, ce, abcde ).

c) Give in detail the analysis of a $2^3$-experiment conducted in an RBD. ( 2 + 2 + 6 )

Problem 14

a) What are the advantages of sample survey over complete enumeration ?

b)If the population consists of a linear trend, then show that $V_{st} \le V_{sys} \le V_{rand}$ . ( 4 + 6 )

Problem 15

a) What is double sampling procedure ? Give an example where the double sampling procedure is used.

b) To get ratio estimator of the population mean double sampling procedure is adopted. Find out approximate variance of the estimator.

c) Introducing appropriate cost function, compare the performance in (b) with that of an estimator based on a single sampling procedure with a specified total cost. ( 2 + 4 + 4 )

Problem 16

Obtain an unbiased estimator of the population total with its variance in a PPSWR sampling. Also find the unbiased estimator of the variance. (6 + 4)

Problem 17

a) What are the advantages of sample survey over complete enumeration ?

b) If the population consists of a linear trend, then show that $V_{st} \le V_{sys} \le V_{rand}$ . ( 4 + 6 )

Problem 18

a) What is double sampling procedure ? Give an example where the double sampling procedure is used.

b) To get ratio estimator of the population mean double sampling procedure is adopted. Find out approximate variance of the estimator.

c) Introducing appropriate cost function, compare the performance in (b) with that of an estimator based on a single sampling procedure with a specified total cost. ( 2 + 4 + 4 )

Problem 19

a) A simple random sample of size 3 is drawn from a population of size N with replacement. Show that the probability that the sample contains 2 different units is $3(N-1)/N^2$.

b) In SRSWOR, show that the sample proportion p is an u.e. of the population proportion P. Also find V(p) and v(p).

( 4 + 6 )

Problem 20

a) When a linear-model is said to be an ANOVA model ? What is 'valid-error' in the context of ANOVA?

b) Discuss, in detail, the analysis of variance technique for one-way classified data under fixed effect model. Derive the expectation of between SS. (2 + 2 + 6)

Problem 21

a) In what respects do analysis of variance, regression analysis and analysis of covariance differ?

b) Stating appropriate assumptions, describe tests based on analysis of variance technique for

i) presence of regression and ii) linearity of regression

in a bivariate set-up where the independent variable is assumed to be non-stochastic. ( 3 + 7 )

Problem 22

a) What is a treatment contrast? b) What are the basic principles of design of experiments? c) Describe the layout and advantages of a RBD. d) Derive the efficiency of a RBD compared to a CRD. ( 1 + 3 + 3 + 3 )

Problem 23

a) What do you mean by a linear model? Explain how the model for one way classification can be considered as a linear model. b) What is local control ? Give an example. c) Workout the analysis of an RBD when two observations are missing. ( 3 + 2 + 5 )

Problem 24

a) Explain the terms 'complete confounding' and 'partial confounding'.

b) Identify the confounded effects of a replicate of ($2^5$, $2^2$) design in blocks of 8 plots, of which the following is a block : ( a, abc, ade, bd, be, cd, ce, abcde ).

c) Give in detail the analysis of a $2^3$-experiment conducted in an RBD. ( 2 + 2 + 6 )

## Paper : IX

Problem 1

a) What do you mean by 'rational sub-groups'?

b) In Shewhart's control chart technique why we take $\mu \pm 3\sigma$ as upper and lower control limits?

c) Distinguish between control limits and specification limits. ( 2 + 4 + 3 )

Problem 2

a) Describe control chart for fraction defective.

b) Construct control chart for number of defects. ( 5 + 5 )

Problem 3

Discuss the following concepts in connection with sampling inspection plan:

i) Consumer's Risk ii) Producer's Risk iii) AOQL iv) OC curve and v) ASN. ( 2 x 5 = 10 )

Problem 4

Describe double sampling inspection plan. Also find consumer's risk and AOQL for this plan. ( 6 + 4 )

Problem 5

Why do we need sequential inference? Describe Wald's SPRT. For the SPRT with stopping bounds A and B ( < A ) and strength ( $\alpha$, $\beta$ ), show that $A \leq (1-\beta)/\alpha$, $B \geq \beta/(1-\alpha)$, $0 < \alpha, \beta < 1$. ( 2 + 4 + 4 )

Problem 6

Suppose the r.v. X follows $b(1,\theta)$. For testing $H_0: \theta = \theta_0$ against $H_1 : \theta = \theta_1$ construct the SPRT. Also find the approximate expressions for the OC and ASN functions. ( 4 + 3 + 3 )

Problem 7

a) If N be the number of observations required for the termination of the SPRT then show that $P(N < \infty) = 1$.

b) Describe the median test. ( 5 + 5 )

Problem 8

Why do we need sequential inference? Describe Wald's SPRT. For the SPRT with stopping bounds A and B ( < A ) and strength ( $\alpha$, $\beta$ ), show that $A \leq (1-\beta)/\alpha$, $B \geq \beta/(1-\alpha)$, $0 < \alpha, \beta < 1$. ( 2 + 4 + 4 )

Problem 9

Suppose the r.v. X follows $b(1,\theta)$. For testing $H_0: \theta = \theta_0$ against $H_1 : \theta = \theta_1$ construct the SPRT. Also find the approximate expressions for the OC and ASN functions. ( 4 + 3 + 3 )

Problem 10

a) If N be the number of observations required for the termination of the SPRT then show that $P(N < \infty) = 1$.

b) Describe the median test. ( 5 + 5 )

Problem 11

a) What do you mean by 'rational sub-groups'?

b) In Shewhart's control chart technique why we take $\mu \pm 3\sigma$ as upper and lower control limits?

c) Distinguish between control limits and specification limits. ( 2 + 4 + 3 )

Problem 12

a) Describe control chart for fraction defective.

b) Construct control chart for number of defects. ( 5 + 5 )

Problem 13

Discuss the following concepts in connection with sampling inspection plan:

i) Consumer's Risk ii) Producer's Risk iii) AOQL iv) OC curve and v) ASN. ( 2 x 5 = 10 )

Problem 14

Distinguish between tolerance limits and confidence limits. Find a distribution free confidence interval for population median. ( 4 + 6 )

Problem 15

Define Pearsonian $\chi^2$ statistic. Discuss how this statistic can be used for testing

i ) homogeneity of several population

ii) independence of two attributes. ( 2 + 4 + 4 )

Problem 16

Describe double sampling inspection plan. Also find consumer's risk and AOQL for this plan. ( 6 + 4 )

Problem 17

Why do we need sequential inference? Describe Wald's SPRT. For the SPRT with stopping bounds A and B ( < A ) and strength ( $\alpha$, $\beta$ ), show that $A \leq (1 - \beta)/\alpha$, $B \geq \beta/(1 - \alpha)$, $0 < \alpha, \beta < 1$. ( 2 + 4 + 4 )

Problem 18

Suppose the r.v. X follows $b(1,\theta)$. For testing $H_0: \theta = \theta_0$ against $H_1 : \theta = \theta_1$ construct the SPRT. Also find the approximate expressions for the OC and ASN functions. ( 4 + 3 + 3 )

Problem 19

a) If N be the number of observations required for the termination of the SPRT then show that $P(N < \infty) = 1$.

b) Describe the median test. ( 5 + 5 )

Problem 20

a) What do you mean by 'rational sub-groups'?

b) In Shewhart's control chart technique why we take $\mu \pm 3\sigma$ as upper and lower control limits?

c) Distinguish between control limits and specification limits. ( 2 + 4 + 3 )

Problem 21

a) Describe control chart for fraction defective.

b) Construct control chart for number of defects. ( 5 + 5 )

Problem 22

Discuss the following concepts in connection with sampling inspection plan:

i) Consumer's Risk ii) Producer's Risk iii) AOQL iv) OC curve and v) ASN. ( 2 x 5 = 10 )

## **Paper : X**

1. Given the following data where $p_0$, $q_0$ and $p_1$, $q_1$ denote price and quantity for base period and current period respectively. Find 'x', if the ratio between Laspeyres'(L) and Paasche's(P) index number is L : P = 28 : 27. 5

| Commodities | $p_0$ | $q_0$ | $p_1$ | $q_1$ |
|---|---|---|---|---|
| A | 1 | 10 | 2 | 5 |
| B | 1 | 5 | x | 2 |

2. Owing to change in prices the consumer price index of working class in a certain area rose in a month by one quarter of what it was before to 225. The index of food became 252 from 198, that of clothing from 185 to 205, that of fuel and lighting from 175 to 195, and that of miscellaneous from 138 to 212. The index of house rent, however, remained unchanged at 150. It known that the weights of clothing, house rent and fuel & lighting were the same. Find out the exact weights of all groups. 10

3. The data regarding the average monthly number of tourists coming to India in different years are given below. Fit an exponential trend curve. What would be the trend value for 1999? 8

| Year | 1990 | 1991 | 1992 | 1993 | 1994 |
|---|---|---|---|---|---|
| No. of tourists | 28038 | 32271 | 34119 | 35367 | 38392 |

4. Suppose a random sample of size 12 (SRSWOR) drawn from a population of 48 individuals gives the following observations ( scores in English ). Give an u.e. of the average score in the population. Also give an estimate of the standard error of the above estimate. 6

42, 28, 17, 24, 24, 15, 37, 42, 23, 44, 37, 30.

5. The no. of persons dying at age 75 is 476 and the complete expectation of life at ages 75 and 76 are 3.92 and 3.66 respectively. Find the no. of persons living at ages 75 and 76 respectively. 5

6. Fill in the blanks in a portion of life-table given below : 6

| Age(yrs.) | $l_x$ | $d_x$ | $p_x$ | $q_x$ | $L_x$ | $T_x$ | $e^o_x$ |
|---|---|---|---|---|---|---|---|
| 4 | 95000 | 500 | ? | ? | ? | 4850300 | ? |
| 5 | ? | 400 | ? | ? | ? | ? | ? |

7. An experiment was conducted to determine the effects of different dates of planting and different methods of planting on the yield of sugar-cane. The data below show the yields of sugar-cane in metric-ton for 4 different dates and 3 different methods of planting. Carry out an analysis of variance for the given data. 10

(Given $F_{0.05;2,6} = 5.14$ and $F_{0.05;3,6} = 4.76$ )

| Method of planting | Date of planting | | | |
|---|---|---|---|---|
| | October | November | February | March |
| I | 7.10 | 3.69 | 4.70 | 1.90 |
| II | 10.29 | 4.79 | 4.58 | 2.64 |
| III | 8.30 | 3.58 | 4.90 | 1.80 |

8. The following table shows the group indices and the corresponding weights for the year 1995, with 1981 as the base ( = 100 ), for a given community:

| Group | Group Index | Weight |
|---|---|---|
| Food | 212.45 | 65.3 |
| Clothing | 328.06 | 4.8 |
| Fuel & Light | 342.89 | 8.5 |
| House Rent | 170.41 | 6.6 |
| Miscellaneous | 203.53 | 14.8 |

i) Find the CLI for the year 1995.

ii) What is the purchasing power in 1995 as compared to 1981?

iii) If Mr. Das's salary increased from Rs. 8000 in 1981 to Rs. 16000 in 1995, how had his economic status changed? ( 4 + 3 + 3 )

9. The data regarding the average monthly number of tourists coming to India in different years are given below. Fit an exponential trend curve. What would be the trend value for 1999? 15

| Year | 1990 | 1991 | 1992 | 1993 | 1994 |
|---|---|---|---|---|---|
| No. of tourists | 28038 | 32271 | 34119 | 35367 | 38392 |

10 . Suppose a random sample of size 12 (SRSWOR) drawn from a population of 48 individuals gives the following observations ( scores in English ). Give an u.e. of the average score in the population. Also give an estimate of the standard error of the above estimate. 10

42, 28, 17, 24, 24, 15, 37, 42, 23, 44, 37, 30.

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| Method of planting | Date of planting | | | |
|---|---|---|---|---|
| | October | November | February | March |
| I | 7.10 | 3.69 | 4.70 | 1.90 |
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| III | 8.30 | 3.58 | 4.90 | 1.80 |

(Given $F_{0.05;2,6} = 5.14$ and $F_{0.05;3,6} = 4.76$ )

## Question Bank for Part – I (General)

1. State the classical definition of probability. Discuss its limitations, giving examples. 2

2. For any two events A and b, show that P (AUB) $\geq$ max {P (A), P (B)}. 2

3. Find the probability that a leap year, selected at random, will have 53 Saturdays.

4. Define moment generating function (m.g.f.) of a random variable. Give example of a distribution whose m.g.f. does not exist. Obtain the m.g.f. of a Poisson distribution with parameter $\theta$. Hence obtain the mean of the distribution. ( 2 + 2 + 3 + 3 )

5. For any two events show that P (AUB) = P(A) + P(B) + P ( A∩B). Hence show that P ( A∩B) $\geq$ P(A) + P(B) – 1. 3

6. If A and B are independent, then show that $A^c$ and B are also independent. 2

7. n cheques drawn in favour of n suppliers are randomly placed in n envelopes addressed to the suppliers. What is the probability that no cheques goes into the right envelope? Also calculate the limiting value of this probability when $n \to \infty$. 5

8. If X has the geometric distribution with parameter p, show that $P(X \geq m + n / X \geq n) = P(X \geq n)$, m,n ε N. Interpret the result.

9. For a r.v. X~ N ( $\mu$, $\sigma^2$ ), prove that $\mu_{2r} = (2r – 1)(2r – 3)\ldots\ldots 5.3.1\ \sigma^{2r}$ where $\mu_{2r}$ is the central moment of order 2r. 5

10. Consider the r.v. X with p.d.f. f(x) = b exp[- b(x - a)]; a < x < ∞, b>0. Show that E $|X – \mu_e|$ = log2 e / b. [$\mu_e$ = median]. 5

## Question Bank for Part – II (General)

1. Examine the characteristics given below and state, in each case, whether it is an attribute or a variable. In the latter case indicate whether it is discrete or continuous :
   mother tongue, annual income of a family and number of workers of a rice mill. 3
2. Define class boundary and cumulative frequency . 2
3. What do you mean by 'ogives' ? How are they constructed ? Mention their uses. 5
4. Describe the different parts of a table. 4
5. What is skewness ? Suggest a measure of skewness based on quartiles. Show that this measure lies between -1 and +1. 6
6. What is ratio chart ? Point out its advantage over a simple line diagram. 2 + 2
7. Show that mean deviation about the median cannot be greater than the standard deviation. 4
8. Find the variance of first n natural numbers. 2

9. For two attributes having two categories each, explain the ideas of independence and association, and hence those of complete and absolute association. 5
10. Let s and R, respectively denote the *standard deviation* and *range* of a set of n values of a variable. Then show that $R^2 / 2n \leq S^2 \leq R^2 / 4$. When do the equalities hold ? 5
11. In a joint study of two variables x and y, paired data on ( x, y ) have been obtained. On the basis of such data obtain a) a LS regression equation of x on y,
    b) the variance of the predicted x-values and
    c) the residual variance. ( 6 + 2 + 2 )
12. State one situation when rank correlation may be used. Derive Spearman's measure of rank correlation coefficient for an untied case. Discuss the cases when it can assume the extreme values. ( 2 + 4 + 4 )

13. Define intra-class correlation coefficient. Derive the formula for intra-class correlation coefficient ($R_I$) when a variate x is measured for p families, each containing k members. Obtain the limits of $R_I$ with their interpretation. ( 2 + 4 + 4 )

14. Answer the following : 1 (each)

a) Mention a method of collecting ***primary data***.

b) When do you prefer a ***ratio chart*** to represent data ?

c) Define ***relative*** frequency.

d) Define ***co-efficient of variation*** (C.V.).

e) Mention a disadvantages of ***harmonic mean*** (H.M.).

f) What is meant by ***median*** of a distribution ?

15. Answer the following : 2 (each )

a) Distinguish between an ***attribute*** and a ***variable***.

b) How is ***Histogram*** drawn for a grouped frequency distribution ?

c) Define an ***ogive***. How is it drawn ?

d) Give an example when **H.M.** is more appropriate compared to **A.M.**

e) Write down the formula for ***mode*** in case of a grouped frequency distribution.

f) If A, G and H be the A.M., G.M. and H.M. respectively of two positive numbers **m** and **n** then show that $\mathbf{G^2} = \mathbf{AH}$.

g) Find the variance of **1$^{st}$ n** natural numbers.

h) Give suitable measure of ***central tendency*** and measure of ***dispersion*** for a frequency distribution with both the terminal classes are open.

i) Show that ***standard deviation*** is independent of any change of origin but dependent on the change of scale.

j) Give a measure of ***skewness*** in terms of $2^{nd}$ and $3^{rd}$ order central moments.

k) Show that $[(n + 1)/2]^{\mathbf{n}} \geq n!$, where n is positive integer.

l) The numbers 3.2, 5.8, 7.9, and 4.5 have frequencies x, (x+2), (x-3) and (x+6) respectively. If their A.M. is 4.876, find the frequencies.